NAG Toolbox for MATLAB

f07he

1 Purpose

f07he solves a real symmetric positive-definite band system of linear equations with multiple right-hand sides,

$$AX = B,$$

where A has been factorized by f07hd.

2 Syntax

```
[b, info] = f07he(uplo, kd, ab, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07he is used to solve a real symmetric positive-definite band system of linear equations $AX = B$, the function must be preceded by a call to f07hd which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If **uplo** = 'U', $A = U^{\mathrm{T}}U$, where U is upper triangular; the solution X is computed by solving $U^{\mathrm{T}}Y = B$ and then $UX = Y$.

If **uplo** = 'L', $A = LL^{\mathrm{T}}$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^{\mathrm{T}}X = Y$.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates how A has been factorized.

uplo = 'U'

$A = U^{\mathrm{T}}U$, where U is upper triangular.

uplo = 'L'

$A = LL^{\mathrm{T}}$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **kd – int32 scalar**

k_d, the number of superdiagonals or subdiagonals of the matrix A.

Constraint: **kd** ≥ 0.

3: **ab**(**ldab**,$*$) **– double array**

The first dimension of the array **ab** must be at least **kd** $+ 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The Cholesky factor of A, as returned by f07hd.

4: **b(ldb,∗) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldb

5.4 Output Parameters

1: **b(ldb,∗) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X.

2: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows: 1: **uplo**, 2: **n**, 3: **kd**, 4: **nrhs_p**, 5: **ab**, 6: **ldab**, 7: **b**, 8: **ldb**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x = b$, where

if **uplo** = 'U', $|E| \leq c(k+1)\epsilon\left|U^{\mathrm{T}}\right||U|$;
if **uplo** = 'L', $|E| \leq c(k+1)\epsilon|L|\left|L^{\mathrm{T}}\right|$,

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(k+1)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \left\|\left|A^{-1}\right||A||x|\right\|_\infty / \|x\|_\infty \leq \mathrm{cond}(A) = \left\|\left|A^{-1}\right||A|\right\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling f07hh, and an estimate for $\kappa_\infty(A)$ $(= \kappa_1(A))$ can be obtained by calling f07hg.

8 Further Comments

The total number of floating-point operations is approximately $4nkr$, assuming $n \gg k$.

This function may be followed by a call to f07hh to refine the solution and return an error estimate.

The complex analogue of this function is f07hs.

9 Example

```
uplo = 'L';
kd = int32(1);
ab    =    [2.343074902771996,    2.078877201506509,    1.130612248337004,
1.146524711734229;
     1.143796127400537, -1.149659055507477, -1.963537900164584, 0];
b = [22.09, 5.1;
     9.31, 30.81;
     -5.24, -25.82;
     11.83, 22.9];
[bOut, info] = f07he(uplo, kd, ab, b)
```

```
bOut =
    5.0000   -2.0000
   -2.0000    6.0000
   -3.0000   -1.0000
    1.0000    4.0000
info =
           0
```